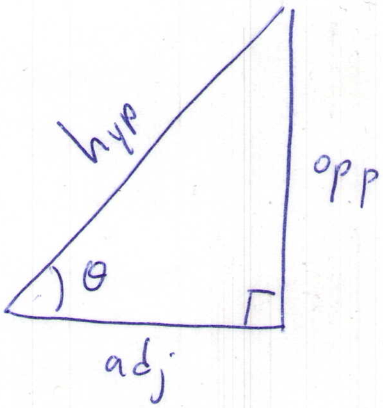
Section 4.3 - Right triangle trigonometry

right triangle = triangle with an angle of $\frac{\pi}{2}$



Definitions:

$$\boxed{\sin\theta = \frac{opp}{hyp}}$$

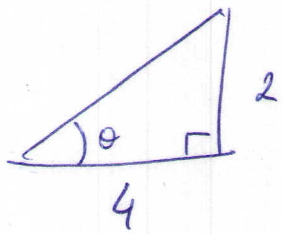
$$\boxed{\cos\theta = \frac{adj}{hyp}}$$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opp}{adj}}$$

Important values

	$\pi/6 = 30°$	$\pi/4 = 45°$	$\pi/3 = 60°$
$\sin\theta$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\cos\theta$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
$\tan\theta$	$\sqrt{3}/3$	1	$\sqrt{3}$

Example: Given the triangle in the picture:



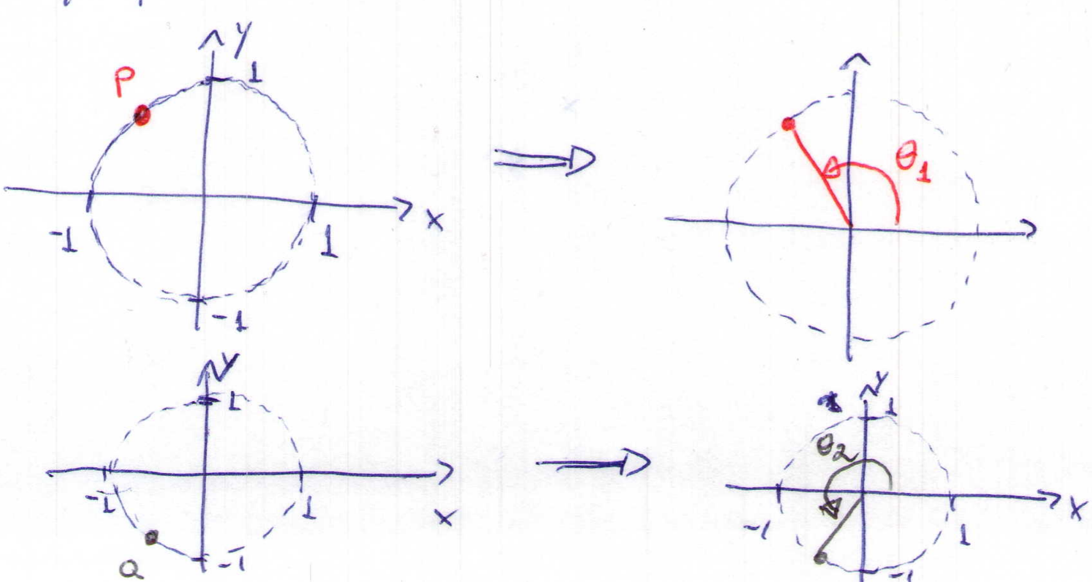
Find $\cos\theta$.

Solution: Denote by h the hypothenuse. Then by the Pythagorean theorem: $h^2 = 4^2 + 2^2 = 16 + 4 = 20$

$\Rightarrow h = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$. So $\cos\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$

Section 4.3 - Trig functions of any angle

We can think of an angle as determined by a point along the circle of radius 1 in the xy-plane:



circle of radius 1

||

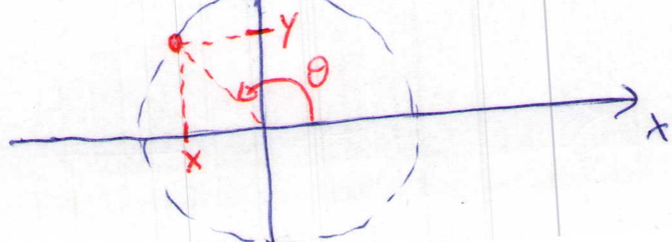
<u>unit circle</u>

In the previous pictures the point P determines an ~~angle~~ angle θ_1 and Q determines an angle θ_2.

And vice-verse: given the angle θ_1 we can find the point P as the point along the circle of radius ~~one~~ 1 which lies on the terminal side of θ_1.

The circle of radius 1 on the xy-plane is sometimes called the <u>trigonometric circle</u>.

So, if we have θ we can think of it as a point at the unit circle; this point has a x-coordinate and a y-coordinate, and for simplicity we will call these the "x and y coordinates of θ". See the picture:

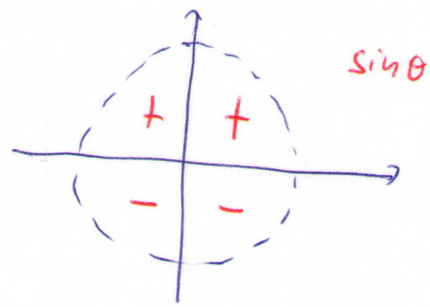


We define:
(see picture in the previous page)

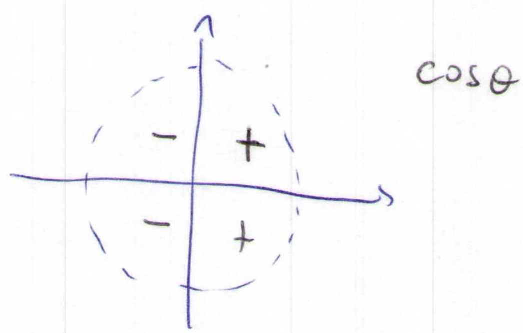
$\boxed{\sin\theta = y}$

$\boxed{\cos\theta = x}$

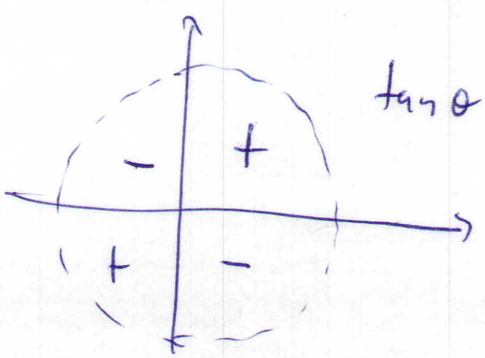
Because y is positive on 1^{st} and 2^{nd} quadrants and negative on the 3^{rd} and 4^{th}, we have that the signs of $\sin\theta$ are:



Because x is positive on the 1^{st} and 4^{th} quadrant and negative on the 2^{nd} and 3^{rd} ones, the signs of $\cos\theta$ are:

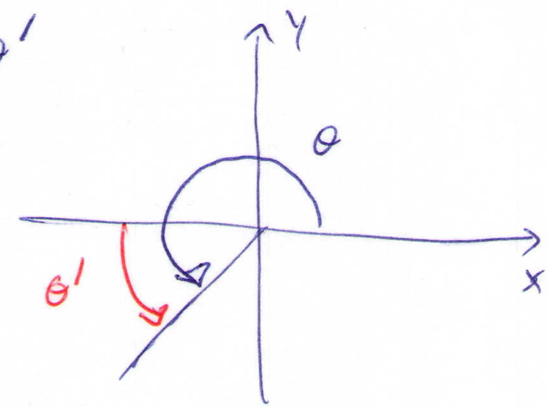


And since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the signs of $\tan\theta$ are:



Finding trig functions of any angle θ

1. Find a <u>positive</u> acute angle θ' formed by the terminal side of θ and the x-axis



2. Compute the trig function of θ' using the chart on page 1.

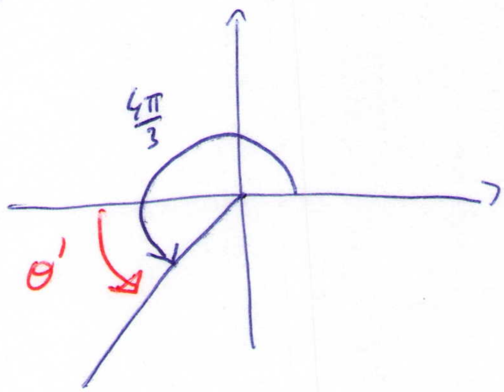
3. Use the signs on page 4 to see if the answer is positive of negative.

<u>Example</u> : Find $\cos\left(\frac{4\pi}{3}\right)$ and $\sin\left(\frac{4\pi}{3}\right)$

It always helps to draw a picture and to do so we need to find the quadrant of $\frac{4\pi}{3}$. Notice that $\frac{4\pi}{3}$ is greater than π (because $\frac{4}{3}$ is greater than one) but less than $\frac{3\pi}{2}$ (because $\frac{4}{3}$ is less than $\frac{3}{2}$), so $\frac{4\pi}{3}$ lies on the 3^{rd} quadrant.

Alternatively, you can use $\pi = 180°$ so $\frac{4\pi}{3} = \frac{4 \times 180°}{3} = 240°$ and $240°$ is on the 3^{rd} quadrant.

We will solve the problem both in rad and degrees:



Since half-circle gives π:

$$\theta' = \frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$$

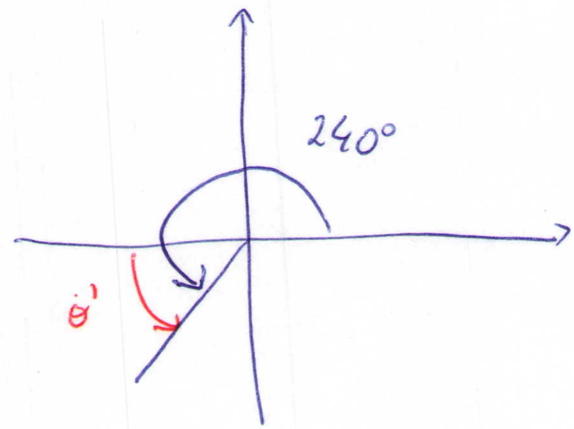
Compute $\cos\frac{\pi}{3} = \frac{1}{2}$

cos is negative on the 3rd quadrant so

$$\cos\frac{4\pi}{3} = -\frac{1}{2}$$

Since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ we get:

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$



Since half-circle gives $180°$:

$$\theta' = 240° - 180° = 60°$$

Compute $\cos 60° = \frac{1}{2}$

cos is negative on the 3rd quadrant so

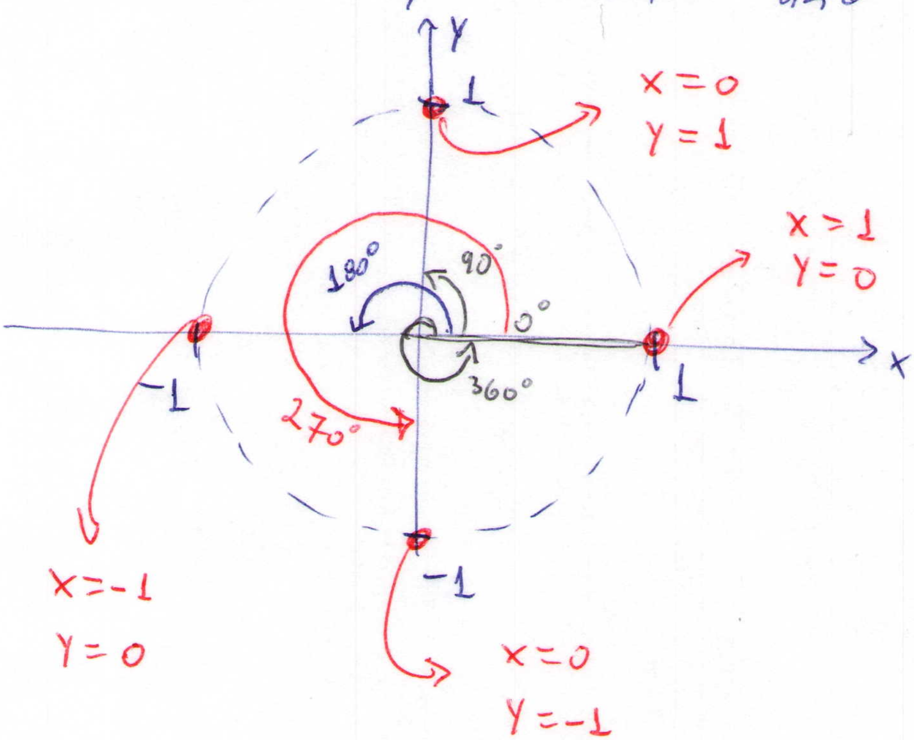
$$\cos 240° = -\frac{1}{2}$$

Since $\sin 60° = \frac{\sqrt{3}}{2}$ we get

$$\sin 240° = -\frac{\sqrt{3}}{2}$$

Values for "special angles": $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

It's very easy to find $\sin\theta$ and $\cos\theta$ for $\theta = 0 = 0°$, $\theta = \frac{\pi}{2} = 90°$, $\theta = \pi = 180°$, $\theta = \frac{3\pi}{2} = 270°$ and $\theta = 2\pi = 360°$. Just draw the unit circle and remember that $\sin\theta = y\text{-coordinate}$ and $\cos\theta = x\text{-coordinate}$



So $\sin 0 = 0$, $\sin 90° = 1$, $\sin 180° = 0$, $\sin 270° = -1$, $\sin 360° = 0$

$\cos 0 = 1$, $\cos 90° = 0$, $\cos 180° = -1$, $\cos 270° = 0$, $\cos 360° = 1$

<u>Remark</u> notice that $\tan 90°$ and $\tan 270°$ are <u>undefined</u> because $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and we cannot divide by zero.