

VANDERBILT UNIVERSITY

MATH 2610 – ORDINARY DIFFERENTIAL EQUATIONS

Examples of sections 1.1 and 1.2

Question 1. Classify the differential equations below as linear or non-linear and state their order.

- (a) $y' + \cos(y) = 0$
- (b) $\frac{d^2x}{dt^2} + \omega^2x = 10 \sin(t)$
- (c) $yy'' + \frac{1}{x} = y$
- (d) $e^{\sin x^2} \frac{dy}{dx} + xy = e^{-x}$

Question 2. A 100 ℓ tank initially contains 10 kg of salt dissolved in 50 ℓ of water. Brine containing 1 kg/ℓ of salt flows into the tank at the rate 2 ℓ/min , and the well-stirred mixture flows out of the tank at the rate 1 ℓ/min . Write an initial value problem for the amount of salt in the tank.

SOLUTIONS.

Question 1.

(a) Non-linear due to $\cos(y)$; first order. (b) Linear second order. (c) Non-linear due to yy'' ; second order. (d) Linear first order.

Question 2. Let $x(t)$ be the amount of salt at time t , measured in kg . Then $\frac{dx}{dt}$ is measured in kg/min , and it is given by

$$\frac{dx}{dt} = in - out.$$

We have

$$in = 1 \frac{kg}{\ell} \times 2 \frac{\ell}{min} = 2 \frac{kg}{min},$$

and

$$out = 1 \frac{\ell}{min} \frac{x(t) kg}{V(t) \ell} = \frac{x(t) kg}{V(t) min}.$$

Since $V(t) = 50 + (2 - 1)t = 50 + t$, we find

$$\frac{dx}{dt} = 2 - \frac{x}{50 + t}.$$

Since at time zero there were 10 kg of salt, the initial condition is $x(0) = 10$. Therefore

$$\begin{cases} \frac{dx}{dt} + \frac{x}{50+t} = 2, \\ x(0) = 10. \end{cases}$$

is the sought initial value problem.