VANDERBILT UNIVERSITY

MATH 2610 – ORDINARY DIFFERENTIAL EQUATIONS

Examples of section 2.4

Question 1. Solve the following IVP:

$$\begin{cases} (4xy + 6y^2)y' + 3x^2 + 2y^2 = 0, \\ y(0) = 2. \end{cases}$$

SOLUTION. Write the equation as

$$(4xy + 6y^2)dy + (3x^2 + 2y^2)dx = 0,$$

and let $M(x, y) = 3x^2 + 2y^2$, and $N(x, y) = 4xy + 6y^2$. Computing we find $\frac{\partial M}{\partial y} = 4y$ and $\frac{\partial N}{\partial x} = 4y$, and therefore this is an exact equation since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Put

$$F(x, y) = \int M(x, y)\, dx = \int (3x^2 + 2y^2)\, dx = x^3 + 2xy^2 + g(y).$$

Then

$$\frac{\partial F}{\partial y} = 4xy + g' = N = 4xy + 6y^2 \Rightarrow g'(y) = 6y^2.$$

Integrating, $g(y) = 2y^3$ (we do not need to add a constant here). Hence the general solution is

$$F(x, y) = x^3 + 2xy^2 + 2y^3 = C.$$

Using $y(0) = 2$ we find $C = 16$, so the solution to the IVP is

$$x^3 + 2xy^2 + 2y^3 - 16 = 0.$$

Notice that this is an implicit solution.