VANDERBILT UNIVERSITY

MATH 2610 – ORDINARY DIFFERENTIAL EQUATIONS

Practice for test 1

The first test will cover all material discussed up to (including) section 4.5.

Question 1. For each equation below, identify the unknown function, classify the equation as linear or non-linear, and state its order.

(a) $y\dfrac{dy}{dx} + \dfrac{y}{x} = 0.$

(b) $x'''' + \cos t\, x' = \sin t.$

(c) $y''' = -\cos y\, y'.$

Question 2. Solve the following initial value problems.

(a) $y' = \dfrac{y-1}{x+3}$, $y(-1) = 0.$

(b) $x' = e^{-t} - 4x$, $x(0) = \dfrac{4}{3}.$

Question 3. Solve the following differential equations.

(a) $y' = \dfrac{\cos y \cos x + 2x}{\sin y \sin x + 2y}.$

(b) $x' = 2t^{-1}x + t^2 \cos t,\ t > 0.$

(c) $x^2 y' = y - 1.$

Question 4. Consider a large tank holding 1000 L of pure water into which a brine solution of salt begins to flow at a constant rate of 6 L/min. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 6 L/min. The concentration of salt in the brine entering the tank is 0.1 kg/L.

(a) Find an initial value problem whose solution gives the amount of salt in the tank at time t.

(b) Solve the initial value problem in (a).

(c) When will the concentration in the tank reach 0.05 kg/L?

Question 5. Find the general solution of the given differential equation.

(a) $x'' + 8x' - 14x = 0$.

(b) $x'' + 8x' - 9x = 0$.

Question 6. Give the form of the particular solution for the given differential equations. You do not have to find the values of the constants of the particular solution.

(a) $x'' + 2x' - 3x = \cos t$.

(b) $x'' + 4x = 8 \sin 2t$.

(c) $x'' - 2x' + x = e^t \cos t$.

(d) $x'' - x' - 12x = 2t^6 e^{-3t}$.

Question 7. Verify that the given functions are two linearly independent solutions of the differential equation.

(a) $x^2 y'' - 2y = 0$, $x > 0$, $y_1 = x^2$, $y_2 = x^{-1}$.

(b) $(1 - x)y'' + xy' - y = 0$, $0 < x < 1$, $y_1 = e^x$, $y_2 = x$.

Question 8. Show that the problem

$$3x' - t^2 + tx^3 = 0,\ x(1) = 6,$$

has a unique solution defined in some neighborhood of $t = 1$.

Question 9. Review the homework problems and examples posted in the course webpage.

Question 10. Know the statement, proof, and how to use the theorems established in class.