MAT 155B - FALL 12 - EXAMPLES OF SECTION 11.4

Determine whether each of the following series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3 + 4n + 3}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{12n^4 + 13n}}$

Solutions. Notice that in all cases the series are of positive terms, hence our usual tests can be applied.

(a). We have

$$\frac{6^n}{5^n - 1} > \frac{6^n}{5^n} = \left(\frac{6}{5}\right)^n.$$

The series

$$\sum_{n=1}^{\infty} \left(\frac{6}{5}\right)^n$$

diverges since it is a geometric series with $r = \frac{6}{5} > 1$, hence

$$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$

diverges by the comparison test.

(b). Notice that

$$\frac{\sqrt[3]{n}}{\sqrt{n^3 + 4n + 3}} < \frac{\sqrt[3]{n}}{\sqrt{n^3}} = \frac{1}{n^{\frac{7}{6}}}.$$

Since

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{7}{6}}}$$

is a p-series with $p > 1$, it converges. Therefore

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3 + 4n + 3}}$$

converges by the comparison test.

(c). Because $\arctan n < \frac{\pi}{2}$, we have

$$\sum_{n=1}^{\infty} \frac{\arctan n}{n^3} \leq \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{1}{n^3}.$$

The series on the right hand side converges (p- series with $p = 3$), so

$$\sum_{n=1}^{\infty} \frac{\arctan n}{n^3}$$

converges as well.

(d). Let us use the limit comparison test. Compute

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt[3]{12n^4+13n}}}{\frac{1}{\sqrt[3]{n^4}}} = \lim_{n\to\infty} \sqrt[3]{\frac{n^4}{12n^4 + 13n}} = \sqrt[3]{12} > 0.$$

Since

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$$

is a p-series with $p = \frac{4}{3}$, it diverges. Therefore

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{12n^4 + 13n}}$$

diverges as well by the limit comparison test.

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`